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INFLUENCE OF CONSTITUENT PROPERTIES  
UPON THE STRUCTURAL EFFICIENCY  
OF FIBROUS COMPOSITE SHELLS

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B.W. ROSEN  
N.F. DOW

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INFLUENCE OF CONSTITUENT PROPERTIES UPON THE STRUCTURAL  
EFFICIENCY OF FIBROUS COMPOSITE SHELLS\*

By

B. Walter Rosen and Norris F. Dow

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## ABSTRACT

The application of fibrous composites to aerospace structures subjected to compressive loads is treated analytically. For boost-vehicle shells, axial compression loadings are found to be low, so that efficiency of filament-reinforced composites for this application depends primarily upon their elastic buckling characteristics. In this study emphasis is placed on the evaluation of elastic shell buckling properties of a wide variety of combinations of filament and binder materials in a number of structural configurations. The influence of filament and binder moduli, filament orientation, hollow filaments, and various effectivenesses of auxiliary stiffening are considered, and the over-all weight-efficiencies are compared with shells made from available structural metals. Results confirm the merits of filament orientations giving isotropic elastic shell properties, advanced filamentary materials like boron, and hollow filaments for unreinforced, monocoque shells. Especially advantageous, and leading to the greatest advances over metallic construction, are combinations with improved binders and highly effective auxiliary stiffening like ideal, light-weight sandwich cores. With such combinations, even at the low loading of boost vehicles, the elastic range can be exceeded, and the high-strength-density ratios of advanced composites may be utilized. For this regime, improved compression failure criteria are needed for final evaluation.

# INFLUENCE OF CONSTITUENT PROPERTIES UPON THE STRUCTURAL EFFICIENCY OF FIBROUS COMPOSITE SHELLS

B. Walter Rosen and Norris F. Dow  
Consulting Engineers, Space Sciences Laboratory  
General Electric Co., Valley Forge, Pa.

## Introduction

The effectiveness of fibrous composites for aerospace structures subjected to compressive loads has been relatively unexplored, yet the potential of even presently available glass-fiber reinforced materials for compressive applications can be assessed as favorable, at least for high loading intensities. Regardless of structural form, for high loading intensities the structure should achieve stresses comparable to the ultimate strength of the material in compression, and the ultimate stress-density ratio of glass-reinforced plastics has been shown (ref. 1, for example) to be substantial. With the advent of filaments like boron with lower densities and higher stiffnesses than glass, the prospects for composites in compression appear favorable also for the lower loadings for which buckling rather than ultimate strength is the design criterion. The purpose of this paper is to explore the potential of filament-reinforced materials for the low compression load intensities appropriate to the shell structure of launch vehicles.

## Approach

To assess the potential of various filamentary materials and binders, a structural-efficiency analysis approach was used. With this analysis the structural weight of an axially compressed cylindrical shell was determined as a function of the prescribed loading conditions. In this section of this report the methods employed in the analysis are described, the range of loading conditions of technological interest for the evaluation are established, the characteristics of the various materials and structural configurations evaluated are reviewed, and some of the reasons for the selection of these particular characteristics for appraisal are cited for guidance in the subsequent interpretations.

## Methods of Efficiency Analysis

The structural efficiency analysis used involved the determination of generalized weights of structural shell required to carry given axial loading intensities. The appropriate parameters for this generalization have been found to be weight per unit surface area divided by shell radius, as a function of axial load per unit length of circumference divided by shell radius. Evaluations of the minimum-weight configuration in each case requires the application of the

appropriate shell failure criteria, which were taken here as either elastic buckling or compressive yielding or fracture.

The elastic buckling portion of the structural efficiency evaluation utilized the small-deflection orthotropic shell stability results of Reference 2, wherein it is shown that the buckling mode is governed by a parameter  $\Phi$ , where  $\Phi = \gamma^{1/2}$  or 1, whichever is smaller, and the shear stiffness ratio,  $\gamma$ , is given by:

$$\gamma = \left[ \frac{G_{LT}}{\frac{\sqrt{E_L E_T}}{2(1 + \sqrt{\nu_{LT} \nu_{TL}})}} \right] \quad (1)$$

with

$G_{LT}$  shear modulus in plane of shell  
 $E_L, E_T$  longitudinal (axial) and transverse (circumferential) stretching moduli of shell  
 $\nu_{LT}, \nu_{TL}$  Poisson's ratios

If  $\gamma > 1$ , the buckling mode is symmetric (bellows-type deformation) and the buckling stress  $\sigma_{cr}$  is given by

$$\sigma_{cr} = \frac{K}{\sqrt{3}} \left( \frac{t}{R} \right) (\bar{E}) \quad (2)$$

where

$$\bar{E} = \left[ \frac{E_L E_T}{1 - \nu_{LT} \nu_{TL}} \right]^{1/2}$$

$\bar{E}$  effective stiffness

$t$  shell thickness

$R$  shell radius

$K$  empirical factor to account for initial imperfections in shell, i.e.  $K \leq 1$ .  
(Herein  $K$  is assumed unity throughout)

If  $\gamma < 1$ , the buckling mode is asymmetric (checkerboard-type deformations) and:

$$\sigma_{cr} = \frac{K}{\sqrt{3}} \left( \frac{t}{R} \right) \left[ 2G_{LT} \left( \frac{\sqrt{E_L E_T}}{1 - \sqrt{\nu_{LT} \nu_{TL}}} \right) \right]^{1/2} \quad (3)$$

The structural efficiency equation employing this expression for elastic buckling is

$$\frac{W}{R} = \rho_s \left[ \frac{\left( \frac{N_x}{R} \right)}{\frac{K}{\sqrt{3}} (\bar{E}) \Phi} \right]^{1/2} \quad (4)$$

where, as before

$$\Phi = \gamma^{1/2} \text{ or } 1 \text{ whichever is the smaller}$$

$N_x$  axial load divided by shell circumference

Equations (2), (3), and (4) apply to simple monocoque shells. As will be shown, for the majority of cases of interest for launch vehicles, stiffened shells are more efficient than monocoque construction. Accordingly, to investigate the potential of fibrous composites for stiffened shells, an idealized stiffening was hypothesized; the shells were assumed made in the form of a sandwich with an ideal core material having adequate stiffness properties through the thickness to stabilize the faces, but having no ability to carry axial load. The elastic buckling efficiency for sandwich shells with such a core is given by (Ref. 2)

$$\frac{W}{R} = \frac{\left[ \rho_s + \rho_c \left( \frac{t_c}{2t_s} \right) \right]}{\left[ \left( 1 + \frac{t_c}{2t_s} \right)^3 - \left( \frac{t_c}{2t_s} \right)^3 \right]^{1/4}} \left[ \frac{N_x}{R} \right]^{1/2} \quad (5)$$

where now

$\rho_s, \rho_c$  densities of face and core materials

$t_s, t_c$  thicknesses of face and core materials

In all cases, minimum-weight sandwich proportions were used, with the optimum ratio of  $\frac{t_c}{2t_s}$  found from the equation

$$\frac{\rho_c}{\rho_s} = \frac{2 \left( \frac{t_c}{2t_s} \right) + 1}{2 \left( \frac{t_c}{2t_s} \right)^2 + 3 \left( \frac{t_c}{2t_s} \right) + \frac{4}{3}} \quad (6)$$

For stresses above the elastic range (as in Ref. 2)

$$\frac{W}{R} = \frac{\rho_s + \rho_c \left( \frac{t_c}{2t_s} \right)}{\sigma_y} \left( \frac{N_x}{R} \right) \quad (7)$$

where

$\sigma_y$  is the compressive "yield" or failure stress for the face materials

As brought out in Reference 3, the determination of a really adequate value of  $\sigma_y$  for fibrous composites is a problem for further research, and conclusions drawn regarding the potential of these materials for those (limited) applications for which equation (7) is used must be somewhat qualified. For launch vehicles, however, as the following sections demonstrate, elastic buckling is the dominant criterion.

#### Evaluation of Elastic Constants

Values of  $E_L, E_T, G_{LT}, \nu_{LT},$  and  $\nu_{TL}$  for use with equations (2), (3), (5), etc. were determined by means of the analytical procedures developed in Reference 4. In brief, the process involves the use of strain energy techniques to obtain bounds on the elastic constants of a unidirectionally reinforced lamina. The bounds coincide for the properties of interest with the exception of the transverse Young's modulus. (What few experimental data there are available, Ref. 5, support these procedures, with the exception of the shear modulus in the plane of the lamina. Measured shear stiffnesses are near the upper bound for the calculations; in consequence the calculated shear properties used here may be somewhat low.) With the elastic constants determined for a unidirectional lamina, the properties of the laminates of the several configurations considered were obtained using the equations of Appendix 3 of Reference 3.

#### Determination of Loadings of Interest for Launch Vehicles

Values of thrust, radius, and thrust per unit circumference and radius for a variety of launch vehicles are given in Table 1. For all boosters considered, the ratio of thrust to the product of circumference and radius is within the range 10 to  $1000 \frac{\text{KN}}{\text{m}^2}$  (1 to 100 psi). Actual design loading intensities  $\frac{N_x}{R}$  are greater than these ratios by factors of 1.5 to 2.0 to account for bending, factors of safety, etc. While these design values of  $\frac{N_x}{R}$  fall toward the higher end of the two-decade range defined above, it appears that shell loading intensities between 10 and  $1000 \frac{\text{KN}}{\text{m}^2}$  (1 and 100 psi) amply cover the range of interest for launch vehicles even including the future Nova-

class boosters. Accordingly the applicability of fibrous composites in this range was considered herein.

### Materials and Configurations Considered

The materials and configurations considered for the launch-vehicle shell application fell into several classes, as follows:

(a) Metals. First a family of metal shells was analyzed to provide a basis for comparison with the composite shells. This metal family comprised a steel, titanium, aluminum, magnesium, and beryllium alloy with the advanced properties postulated in Table 2. These properties were deliberately chosen to be high relative to present technological values to insure a high standard for the comparisons with composites.

(b) Filaments. A family of eight filamentary materials was selected for use in the composites. These materials began with the presently used E-glass in both solid and hollow fibers and ranged upward in characteristics, including,

High-Modulus Glass  
Asbestos  
Steel  
Beryllium  
Boron  
and Alumina

The properties used for these various filamentary materials are given in Table 3.

(c) Binders. A family of eight binder materials was also selected into which the various filaments were incorporated. The binders began with the presently used epoxy resin and ranged upward in properties, as follows:

Magnesium  
Three hypothetical "Light Alloys"  
Titanium  
Steel  
Boron

The properties used for these various binder materials are also listed in Table 3.

(d) Configurations. All shell composites were considered to be laminates with each lamina unidirectionally reinforced by the filamentary material. The directions of reinforcement of the laminae were varied in symmetric fashion such that the principal stiffnesses of the laminate always coincided with the axial and circumferential shell directions. The number of laminae was supposed great enough so that the laminate acted like a homogeneous medium, -i.e., no attempt was made to dispose internal and external laminae in different fashions. Thus typical configurations included: (1) longitudinal reinforcement;

(2) transverse reinforcement; (3) longitudinal and transverse reinforcement; (4) reinforcement at equal angles (-) to the longitudinal or transverse directions; and (5) three way ( $\pm 30^\circ$ ,  $90^\circ$ ) reinforcement to provide in-plane isotropy.

### Bases for Choice of Materials Used

In order to help isolate the importance of various factors on the end efficiency of composites, the filamentary and binder materials were selected to provide several interrelated systematic types of variations. These inter-relationships are indicated on Figure 1. The matrix material properties plotted in Figure 1a show that:

(1) The magnesium, titanium, and steel materials provide a variation in modulus at constant modulus-to-density ratio.

(2) The magnesium, "Light Alloy II," "Light Alloy III," and boron materials provide a variation in modulus at constant density.

(3) The "Light Alloy I," "Light Alloy II", and titanium provide a variation in density at constant modulus, as do the "Light Alloy III", and steel at a different constant modulus.

The fiber materials were selected to cover the range of actual prospects from those now in actual use like E-glass to those which are more in the nature of laboratory curiosities like alumina. (The fiber properties are presented in Figure 1b.) Thus approximately the entire spectrum of properties of current interest is surveyed as well as those portions of the spectrum which provide systematic variations.

### Results and Discussion

The results of the evaluations of composite shell efficiencies are discussed here in five sections. In the first section the interplay between the magnitude of the design loading and the structural configuration employed to support the load is considered, to insure that the results are not inequitably influenced by the configurations chosen. In the second section, the conclusions drawn from this consideration of the significance of configuration are focussed upon the evaluation of hollow fiber reinforcement. Here some of the aspects of the importance of fiber stiffness are first demonstrated, and then they are examined in detail in the third section. Fourth, the importance of binder stiffness is assessed, and in the final section the needs for improved failure criteria are presented.

#### Effects of Configuration

##### Metal Shells

The basis used for the evaluations of fibrous composite shells is established in Fig. 2. Here are plotted the weights of cylindrical shells of a

wide range of types of metals, and fabricated in a variety of sandwich proportions, designed to carry intensities of loadings of from a small fraction of-to many times those appropriate for launch vehicles, as shown. Characteristically a shell of any material is heavier than the weight required to carry the design load at the material yield stress (represented by the lines of 45° slope at the right of Fig. 2) by the weight of the additional material needed to stabilize the shell against buckling. Generally speaking the greatest weight is required with the stiffening added simply as increased shell thickness, giving a pure monocoque construction (the upper curves on the figure). For the monocoque buckling below the yield stress, the weight is proportional to  $\frac{\rho}{\sqrt{E}}$  as is well known, and beryllium is a currently recognized minimum-weight metal for the idealized monocoque shell, - especially in the low loading intensity regime of interest for launch vehicles.

Clearly, however, Fig. 2 shows that for the metal shell more is to be gained by a change from a monocoque to an efficient stiffening configuration like a low-density-core-material sandwich than by the use of even such an efficient material as beryllium. In fact a steel faced sandwich with a light core may be lighter than a beryllium monocoque shell, -indeed will be lighter than a beryllium faced sandwich on the same core if the core densities are low enough or the loading intensities high enough (the area shown to the right of Fig. 2) so that the higher strength-to-weight ratio of steel compared to beryllium can be utilized.

Important to the composite evaluations to which this study is directed is the implication of the preceding paragraph that the optimum material depends upon both loading that must be carried and upon the structural configuration employed. In consequence both the range of loadings and range of configurations of interest must be surveyed for proper assessment of the potentials of composites. Herein the effect of varying overall configuration is determined by a variation in the hypothetical sandwich core density. The results of this variation are generally the same as variations in the effectiveness of other types of stiffening. Thus, a very light weight core represents to a degree, for example, very efficient integral ribbing on the shell, or highly efficient ring-stringer reinforcement.

One further aspect of the importance of configuration in the evaluation of material efficiency is brought out by the reference shell efficiencies calculated for the variables included in Fig. 2. Whereas for the elastic monocoque shell the weight is proportional to  $\frac{\rho}{\sqrt{E}}$  as previously noted, this relationship does not apply for sandwich shells even for the heaviest core density given in Fig. 2. For values of the ratio of face sheet to core densities large compared with unity,

it can be shown that the shell weight for a given core density is proportional to  $\sqrt{\rho/E}$ . The shell weight required in the elastic buckling range may be measured simply by

$$F = \frac{\frac{W}{R}}{\sqrt{\frac{N_x}{R}}} \quad (8)$$

where  $F$  is a function of shell moduli and density as seen from Figs. (5) and (6). Values of  $F$  are plotted in Fig. 3 for monocoque and sandwich shells of the five metals used in Fig. 2. As shown, values of  $F$  for the monocoque configuration plot on the expected straight line of 45° slope when the abscissa is  $\rho/\sqrt{E}$ , but for the sandwich the abscissa must be changed to  $\left(\frac{\rho}{E}\right)^{1/2}$ . Thus even for elastic buckling the configuration affects the relationship between material properties and shell efficiency, though perhaps not as profoundly as when a change from elastic to plastic behavior is involved.

### Composite Shells

With fibrous composites additional degrees of freedom are available compared to metal construction. Not only may each reinforcing material be employed in a variety of binder materials, but also various volume fractions of the constituents and fiber orientations may be used. To systematize the interpretations from such an array of variables the following sequence will be used in this report. First effects of filament orientation and of filament/binder concentrations as related to the configurational effects of monocoque and sandwich construction will be considered in this section. The influence of fiber geometry and fiber and binder properties will be reported separately in subsequent sections of the report.

#### (a) Filament orientation

For maximum ultimate strength in compression all filamentary reinforcement should be aligned in the axial load direction. For maximum resistance to elastic buckling, on the other hand, an isotropic laminate (Ref. 2) is needed. Thus the best orientation depends upon whether for the given composite the elastic buckling or ultimate strength regime is of prime importance. As will be shown in the following example, no general conclusion can be drawn here as to which of these governs: the specific composite combination must be investigated in each case.

For example consider three possible composites all made with 30 volume percent epoxy binder reinforced respectively with steel, boron, and alumina filaments. The efficiencies of these in various configurations are presented in Figures 4 and 5. In Figure 4 the weights of steel reinforced



epoxy are shown to be greater than the weights of metal shells previously calculated, for all configurations over the entire two decade range of loads found of interest for launch vehicles-with one exception. If an extremely low density (but still structurally sound) sandwich core is available,  $0^{\circ}$  steel reinforced faces will be superior to sheet-metal faces at the higher load intensities (because of the assumed high strength of filamentary steel composites). The isotropic configuration for this particular composite is never in contention. For advanced filamentary materials like boron or alumina, on the other hand, the isotropic pattern is lighter over almost the entire range considered. Thus reinforced, even an unadvanced binder like epoxy is competitive with metals in all configurations considered, and the composites are superior to the best metal shells in the most structurally efficient cases (the lighter core densities and higher loading intensities). Here the significance of configuration to the assessment of material potential is underlined; that is, if the structural configuration is ineffective (the lowly loaded monocoque shell) greater gain can be effected by configuration than by material improvement. If the configuration is good, better materials will enhance the efficiency further.

#### (b) Filament/binder concentrations

The 30% volume fraction binder used in the foregoing examples was only arbitrarily selected as representative of present technology. An investigation of the effect of binder content is therefore in order. Typical effects of binder content are illustrated in Figs. 6 and 7. Fig. 6 illustrates the change in ultimate compressive stress/density potential with binder content, (as presented in Reference 3) and so is applicable at high loadings or low sandwich-core densities. Fig. 7 shows the elastic buckling efficiencies at a typical sandwich core density, and hence applies to the low load end of the range. The variations in stress-density ratios with binder content plotted in Figure 6 reveal no surprises except possibly that the asbestos appears somewhat relatively better in the isotropic than in the uniaxial orientation. Perhaps of greater import is the fact that the ratio between the strength/density ratios for the isotropic and the uniaxial configurations is small at all binder contents. Thus, the penalty paid in ultimate strength by the use of the isotropic arrangement is substantial, and for the higher loading intensities may have to be avoided. Variations in elastic buckling efficiency as measured by the values of  $F$  plotted in Figure 7 on the other hand are rather surprising in that relatively small concentrations of the high-modulus filaments are sufficient to produce materials with buckling effectiveness comparable to structural metals. Accordingly, for the very light loading for which strength is not important, low volume fractions of advanced filaments may be of interest.

Disappointingly inefficient on both plots, Figures 6 and 7, are the results for the hollow E-glass filaments. These results have sufficiently sweeping implications regarding geometrical effects to warrant special attention, and some of these implications are therefore reviewed in the following section on the effects associated with the use of hollow fibers.

#### Effects of Hollow Fibers

Hollow fibers provide a reduction in density,  $\rho_s$ , of composite material together with a high ultimate compressive stress-density ratio (Ref. 6). At the same time the hollows reduce the elastic stiffnesses and the absolute values of strength. Because of these reductions, with E-glass, as pointed out in the preceding section, for sandwich shell faces the hollows are not effective, and indeed compared to metal shells (Fig. 8), hollow E-glass reinforcements appear attractive only for monocoque shells.

Factors which combine to make hollow E-glass ineffective are (1) the fact that for a sandwich face material, density is not as important a characteristic as for a monocoque shell, and (2) the low transverse stiffness properties calculated for the hollow fibers. As previously pointed out, for a sandwich the efficiency varies inversely as only the square root of the face material density, where for the monocoque the efficiency varies as the inverse first power of the density. The low transverse stiffness found for the hollow glass is a less obvious problem, however, that deserves further consideration. Perhaps, for example, the analysis of the hollow fiber transverse stiffness is more open to question than the solid fiber calculations. For both types of fibers the values used are the average of upper and lower bounds but in the case of the solids the bounds are not so far apart that this procedure is open to substantial variation no matter which bound is the more applicable. For the hollows, on the other hand the upper-bound transverse stiffness at 30% binder is approximately twice the value for the lower bound. Thus, the hollow fibers may be appreciably better (or even worse) than the mean value indicates.

Regardless of the accuracy of analysis, perhaps of greater significance for future prospects is the fact that stiffer materials than E-glass should perform better transversely in an epoxy matrix. This effect is illustrated in Figure 9 where the calculated ratios of transverse stiffness (upper and lower bounds) for hollows and solids are plotted against volume fraction of binder for alumina and E-glass fibers. The curves for the alumina are above those for E-glass over the entire range of concentrations, being about twice as great at the normal 30 volume percent binder. In other words not only is the transverse stiffness inherently higher for alumina than E-glass reinforcement, but also the hollow alumina performs twice as well compared to solid as the E-glass does. In sum, hollow E-glass fibers in epoxy binder appear

promising only for increasing the efficiency of shells in applications for which material density is of prime importance, as for monocoque construction. Hollow filaments of higher modulus than E-glass may be relatively more favorable.

Two additional aspects of hollow fibers warrant mention. The first relating to performance at high loading intensities will be covered in the final section of this report. The other pertains to the "minimum gage" problem. Here, for loads so low that the material thicknesses need to be thinner than can be practicably produced, is another area for which density is of prime importance, and for which hollow fibers are indeed appropriate.

#### The Importance of Fiber Stiffness and Density

That the use of high modulus/density ratio filaments like boron should increase the elastic buckling efficiency of composite shells is to be expected, and the effect is demonstrated by the values of  $F$  given in Figure 7 for sandwiches and Figure 10 for monocoque construction. In order to determine just how effective improvements in filaments may be, the filament density and modulus will be treated separately. The face sheet density is related to the fiber and binder densities by a simple mixtures rule. When the fiber weight is a large fraction of the composite weight, as it is for high volume fractions of most fiber materials, then the variation of  $F$  with fiber density is of essentially the same form as that of the variation with face density (i.e., with  $\rho_f$  for monocoque and  $\rho_f^{1/2}$  for sandwich construction). The variation of weight with modulus is most readily studied by plotting  $F$  as a function of  $E_f$  for the family of constant density fibers shown in Figure 1b. For example, Figure 11 presents such results for sandwich shells having isotropic and uniaxial face sheets. The slope of the best fit straight lines can be used to determine the exponent of the fiber modulus in the assumed weight variation:

$$F = K \rho_f^m E_f^n \quad (9)$$

These results are combined to yield the results shown in Figure 12. The exponents have been rounded off to fractional powers as greater accuracy is certainly not justified at the present. Similar results are presented in Figure 13 for the monocoque shell. Correlation of the data are indicated by comparison with curves of 45° slope on the log-log plots of the figures. Approximate correlation is found if the following powers of Young's modulus are employed:

$$\begin{aligned} (1) \quad F &\sim \frac{1}{E_f^{1/6}} \text{ for } 0^\circ \text{ reinforced monocoque shells} \\ (2) \quad F &\sim \frac{1}{E_f^{1/3}} \text{ for Isotropic monocoque shells} \end{aligned}$$

$$(3) \quad F \sim \frac{1}{E_f^{1/6}} \text{ for } 0^\circ \text{ reinforced sandwich shells}$$

$$(4) \quad F \sim \frac{1}{E_f^{1/2}} \text{ for Isotropic sandwich shells}$$

(Correlation is established as for the metal shells by comparison with curves of 45° slope on the log-log plots of the figures.)

Thus it appears that the elastic buckling efficiency of composite shells is a rather insensitive function of the modulus of the fibers. For the configuration of greatest probable interest, however, (the isotropic sandwich), the insensitivity is least, and in this case an increase in fiber modulus is nearly as effective as a decrease in fiber density.

Throughout this section only epoxy binder at 30% volume fraction has been considered. Effects of binder changes will be considered in the following section.

#### The Importance of Binder Stiffness and Density

The use of improved binder material compared to epoxy resin can have several important effects. First, by enhancing the transverse and shearing stiffnesses, it can reduce the difference in buckling efficiencies of the 0° and Isotropic reinforcement configurations. This effect is illustrated in Figure 14.

In Figure 14 are plotted the buckling effectivenesses of composite sandwiches made with very high modulus filaments like alumina embedded in a variety of metallic binders. The plots show that for all binder volume fractions there is little difference in efficiency for the 0° and Isotropic configurations, although the Isotropic cases are always the lighter. For comparison, the  $F$  value for beryllium sandwiches is also given on the figure; for purely elastic buckling it is exceeded in efficiency only at high volume fractions by the boron-like binder. However, it should be recalled that, even for the low loading intensities of launch vehicles, such beryllium sandwiches would be stressed beyond the elastic limit, and any of the alumina-reinforced composites would therefore be more efficient in application (c.f. Fig. 5).

The effectiveness of improvements in binder material properties is similar to that of filament properties. Even with an advanced filament like boron, the buckling efficiency is improved compared to epoxy binder by either an increase in binder modulus or a decrease in binder density (see Fig. 15). Evaluation of the magnitudes of the improvements (Fig. 16) show that they depend only on the one-sixth power of the modulus, for monocoque rather than sandwich shells again the density

is of greater significance, and  $F \sim \sqrt[3]{\frac{\rho_b}{E_b}}$ , approximately.

## The Need for Improved Failure Criteria

The vast majority of the evaluations made in this study involved the elastic buckling properties of composites. As shown in Fig. 2 and Table 1, these are the properties of prime concern for launch vehicles if reasonable stiffening efficiencies (comparable to sandwich core densities greater than  $0.277 \text{ Mg/m}^3$  ( $0.001 \text{ pci}$ ) are assumed. Obviously, however, minimum-weight structures demand optimum stiffening efficiencies, stresses beyond the elastic range, and consequent analysis of all possible failure modes, -with attendant improvement of the criteria for failure.

While the study of the many failure characteristics of composites is beyond the scope of the present paper, one example of the importance of such a study derives directly from the analyses made herein. This example measures the maximum shear stresses developed in composites when assembled in the isotropic laminate configuration found most effective for shell buckling resistance herein. Magnitudes of these stresses are plotted in Figure 17. Figure 17 shows that maximum shear stresses developed with high-modulus fiber reinforcements are substantial. While they are reduced by improved binder stiffness and by high volume fractions of either binders or filaments, they may be sufficient to initiate a premature failure in many compositions.

## Conclusions

The conclusions derived from this investigation of the influence of constituent properties upon the efficiency of composite shells are brought out in the appropriate sections of this report. They are consequently re-stated here in the order in which they were reported, as follows:

1. Loading intensities for launch vehicles are so low that elastic buckling governs the compression design for all but the most efficient stiffening configurations.

2. For sandwich construction the elastic shell buckling efficiency is no longer proportional to the ratio of shell density to the square root of Young's modulus  $\frac{\rho}{\sqrt{E}}$  as for a monocoque shell, but rather is proportional to  $\sqrt{\frac{\rho}{E}}$  for the sandwich face material.

3. Composites reinforced in an isotropic laminate configuration by advanced filaments like boron and alumina are superior to the best metal shells for the most structurally efficient applications for launch vehicles.

4. Relatively small concentrations of high-modulus filaments in an isotropic configuration produce materials with buckling effectiveness comparable to structural metals.

5. Hollow fibers appear promising only for shell buckling applications for which density is of prime importance, (for example; monocoque shells minimum-gage cases).

6. The relation between shell buckling efficiency and filament properties varies with configuration. In general the efficiency is a weak function of the filament modulus and a stronger function of filament density. For the most efficient configuration (isotropic laminate sandwich) the efficiency is proportional to the square root of the filament density and slightly less than the square root of the inverse of the modulus.

7. The relation between shell buckling efficiency and binder properties is similar to that for filaments but is an even weaker function. Thus, for the isotropic sandwich shells the efficiency is approximately proportional to the one-sixth power of the density/modulus ratio.

8. Failure criteria for composite shells need further intensive investigation. Such problems as those of maximum shear stresses in laminates in compression need evaluation.

## References

1. Fried, N. & Winas, R. R., "Compressive Strength of Parallel Filament Reinforced Plastics: Development of a New Test Method," ASTM Tech. Pub. No. 327, Phila., Pa., 1963.
2. Dow, N. F. & Rosen, B. W., "Structural Efficiency of Orthotropic Cylindrical Shells Subjected to Axial Compression," AIAA Preprint 65-73, Jan. 1965.
3. Dow, N. F. & Rosen, B. W., "Evaluations of Filament-Reinforced Composites for Aerospace Structural Applications," Annual Report NASA Contract NASw-817, Oct. 1964.
4. Hashin, Z. & Rosen, B. W., "The Elastic Moduli of Fiber Reinforced Materials," Journal of Applied Mechanics, June 1964.
5. Tsai, S. W., "Structural Behavior of Composite Materials," NASA CR-71, July 1964.
6. \_\_\_\_\_, "Hollow Glass Fiber Reinforced Laminates," Final Report, Contract N0w-63-0674-c, General Electric Company, Aug. 1964.

TABLE 1. COMPRESSIVE LOADINGS FOR LAUNCH VEHICLES

Vehicle	Thrust, kN (lbs.)	Radius, m. (in.)	Thrust
			Circumference x Radius KN/m <sup>2</sup> (psi)
Redstone	347 (78,000)	0.889 (35)	70 (10)
Scout	383 (86,000)	0.4955 (39)	250 (36)
Thor	756 (170,000)	1.219 (48)	80 (12)
Atlas	1730-375* (389,000-80,000)	1.524 (60)	120-24 (17-3.5)
Minuteman	756 (170,000)	0.9015 (35.5)	150 (21.5)
Titan I	1334 (300,000)	1.524 (30)	90 (13)
Titan II	1913 (430,000)	1.524 (30)	130 (19)
Saturn V	33,360 (7,400,000)	5.08 (200)	200 (30)
Nova	11,200 (25,000,000)	12.19 (480)	120 (17)

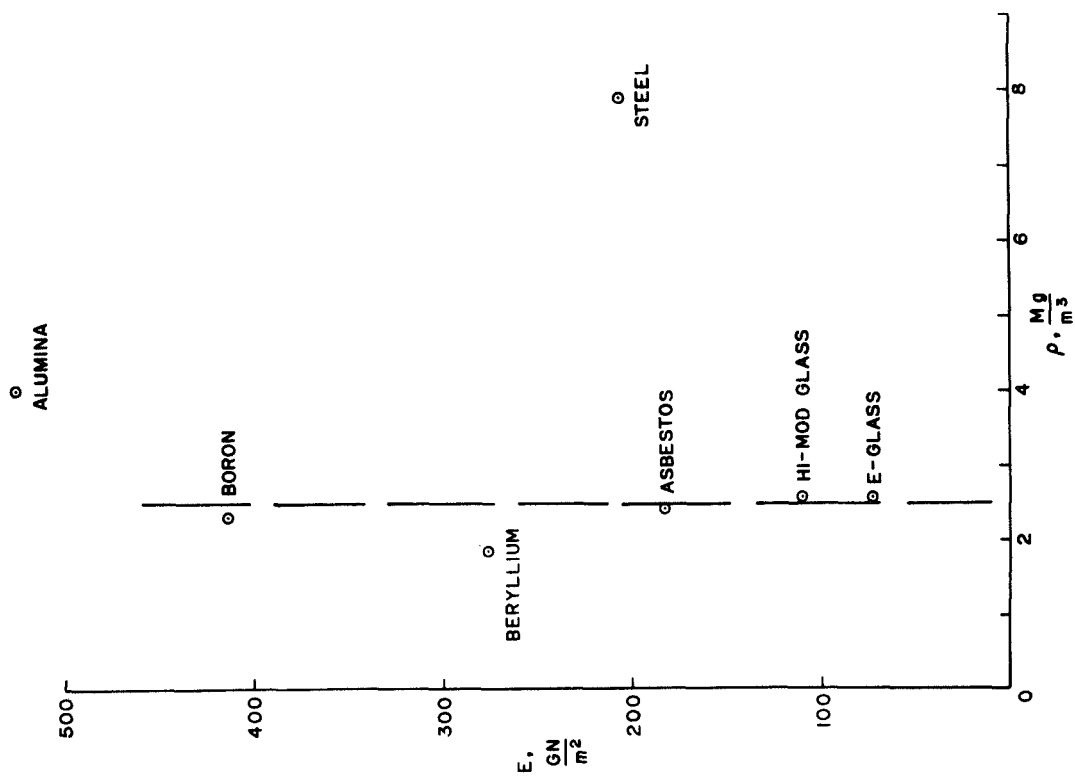
\*Lower value is that for sustainer engine-in this case perhaps more representative of the design condition.

TABLE 2. MECHANICAL PROPERTIES ASSIGNED TO IDEALIZED METALS FOR COMPARISON WITH COMPOSITES

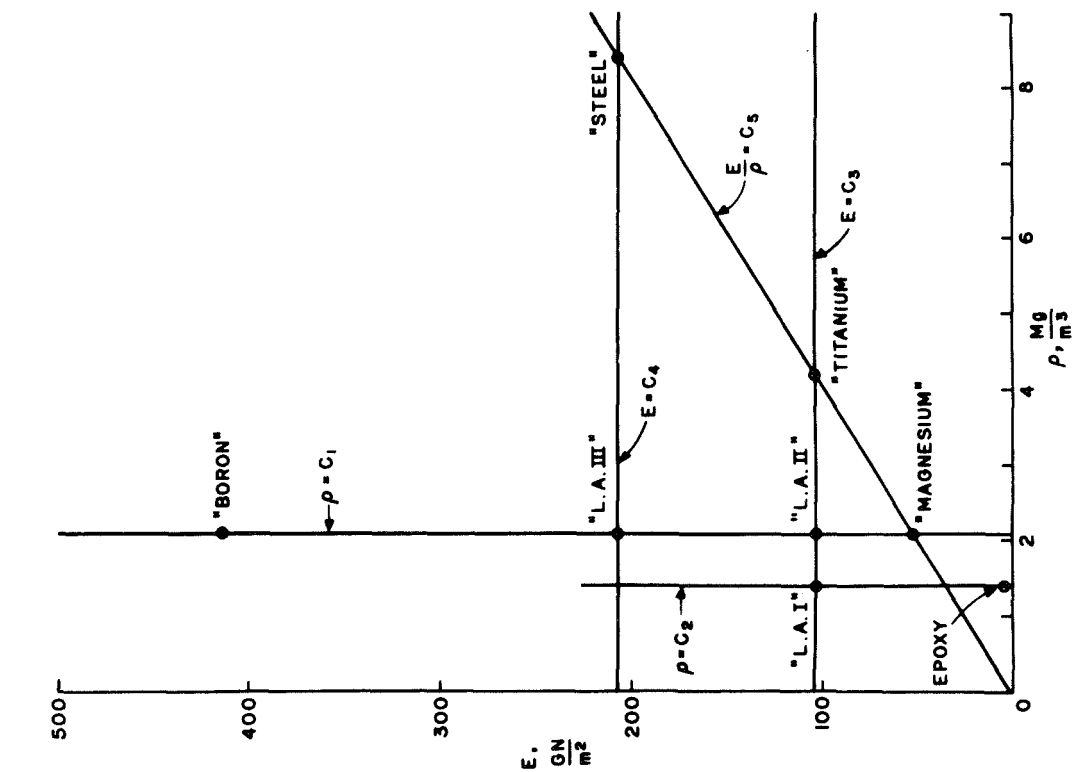
Material	Density Mg/m <sup>3</sup> (pci)	Young's Modulus GN/m <sup>2</sup> (ksi)	Yield Stress GN/m <sup>2</sup> (ksi)	Poisson's Ratio
Steel	7.89 (0.285)	207 (30,000)	2.07 (300)	0.25
Titanium	4.82 (0.174)	103 (15,000)	1.38 (200)	0.145
Aluminum	2.80 (0.100)	73.8 (10,700)	0.483 (70)	0.315
Magnesium -Lithium	1.34 (0.0485)	42.75 (6200)	0.124 (18)	0.43
Beryllium	1.83 (0.066)	293 (42,500)	4.00 (58)	0.09

TABLE 3. MECHANICAL PROPERTIES USED FOR FILAMENTARY AND BINDER MATERIALS  
SURVEYED FOR COMPOSITES

	Young's Modulus GN/m <sup>2</sup> (ksi)	Density Mg/m <sup>3</sup> (pci)	Poisson's Ratio
<u>Filaments</u>			
Hollow E-Glass	72.45 (10,500)	2.56 (0.0914)	0.20
Solid E-Glass	72.45 (10,500)	2.56 (0.0914)	0.20
Hi-Modulus Glass	110 (16,000)	2.56 (0.0914)	0.20
Asbestos	183 (26,500)	2.44 (0.087)	0.20
Steel	207 (30,000)	7.9 (0.283)	0.25
Beryllium	276 (40,000)	1.85 (0.066)	0.09
Boron	414 (60,000)	2.32 (0.083)	0.20
Alumina	518 (75,000)	4.0 (0.143)	0.20
<u>Binders</u>			
"Epoxy"	3.45 (500)	1.40 (0.050)	0.35
"Light-Alloy I"	103.5 (15,000)	1.40 (0.050)	0.30
"Magnesium"	51.75 (7500)	2.10 (0.075)	0.30
"Light-Alloy II"	103.5 (15,000)	2.10 (0.075)	0.30
"Light-Alloy III"	207 (30,000)	2.10 (0.075)	0.30
"Boron"	414 (60,000)	2.10 (0.075)	0.30
"Titanium"	103.5 (15,000)	4.20 (0.150)	0.30
"Steel"	207 (30,000)	8.40 (0.300)	0.30



(a) Matrix Materials



(b) Fiber Materials

Figure 1. Moduli and Densities of Matrix and Fiber Materials Studied

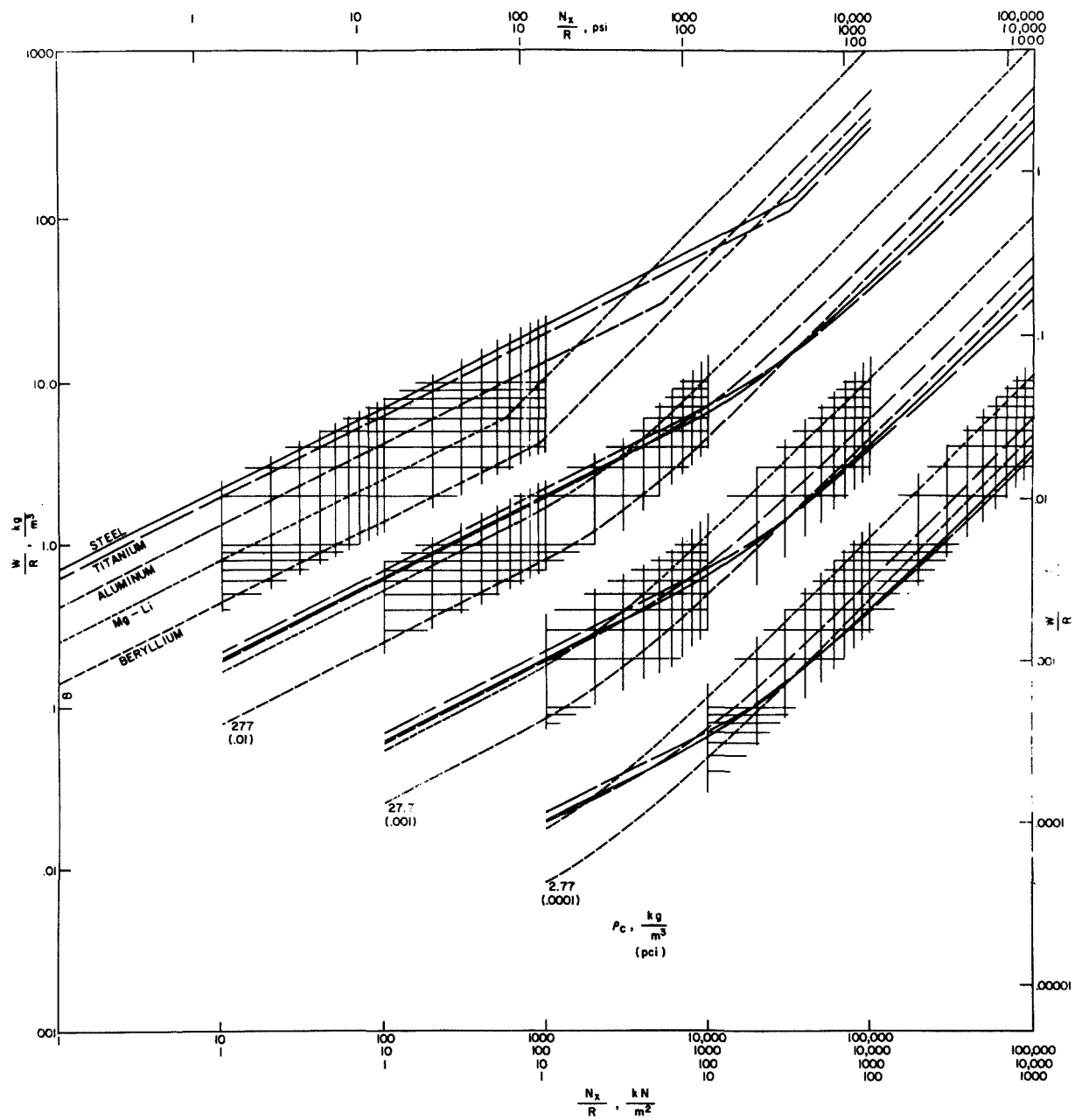


Figure 2. Reference Weight-Efficiencies of Idealized, Metal-Faced Sandwich, Cylindrical Shells for Various Intensities of Axial Loading and Various Densities of Hypothetical, Ideal Core Material

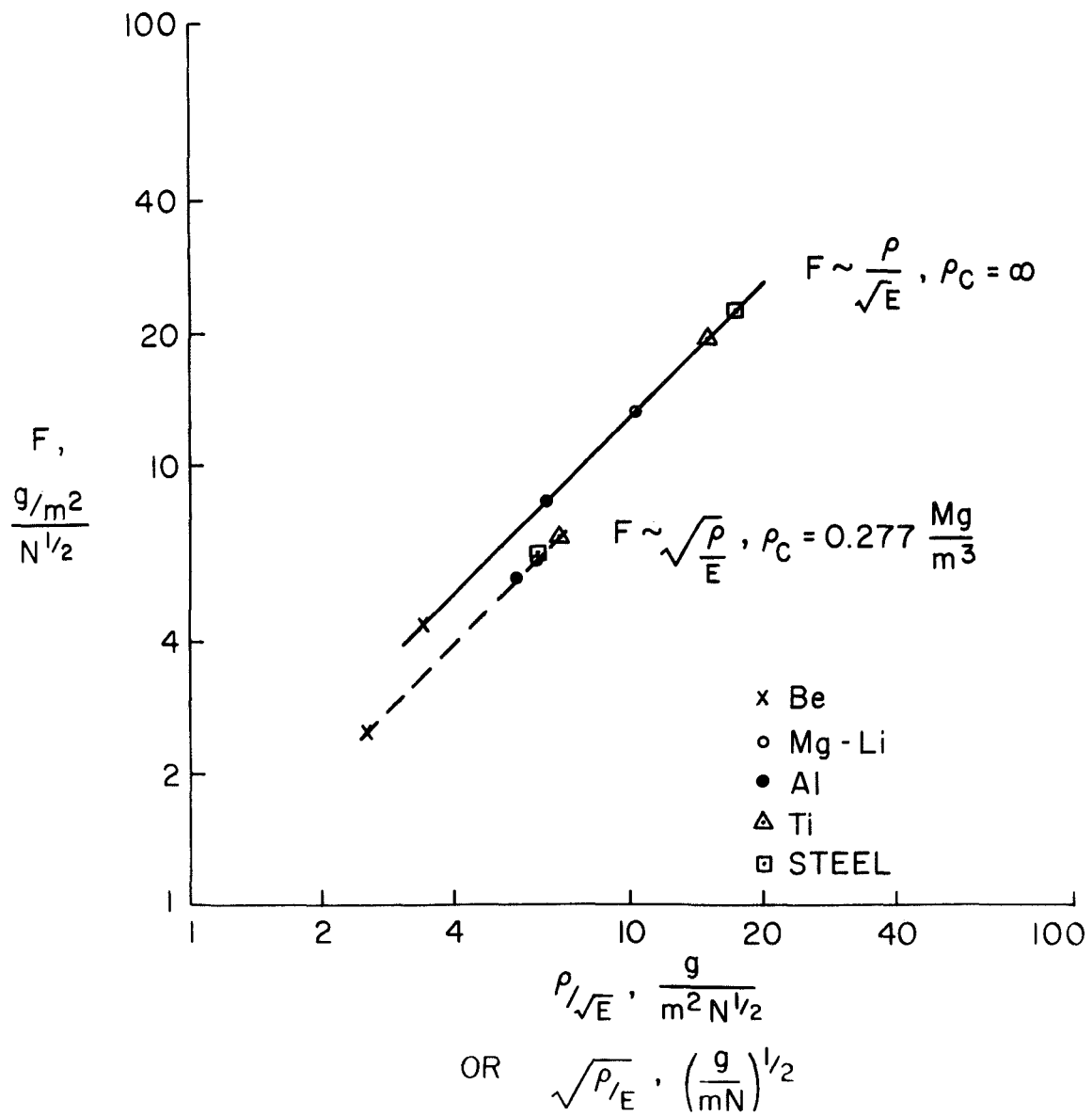


Figure 3. Elastic Buckling Efficiency of Metallic Monocoque and Sandwich Shells as a Function of Shell Material Properties



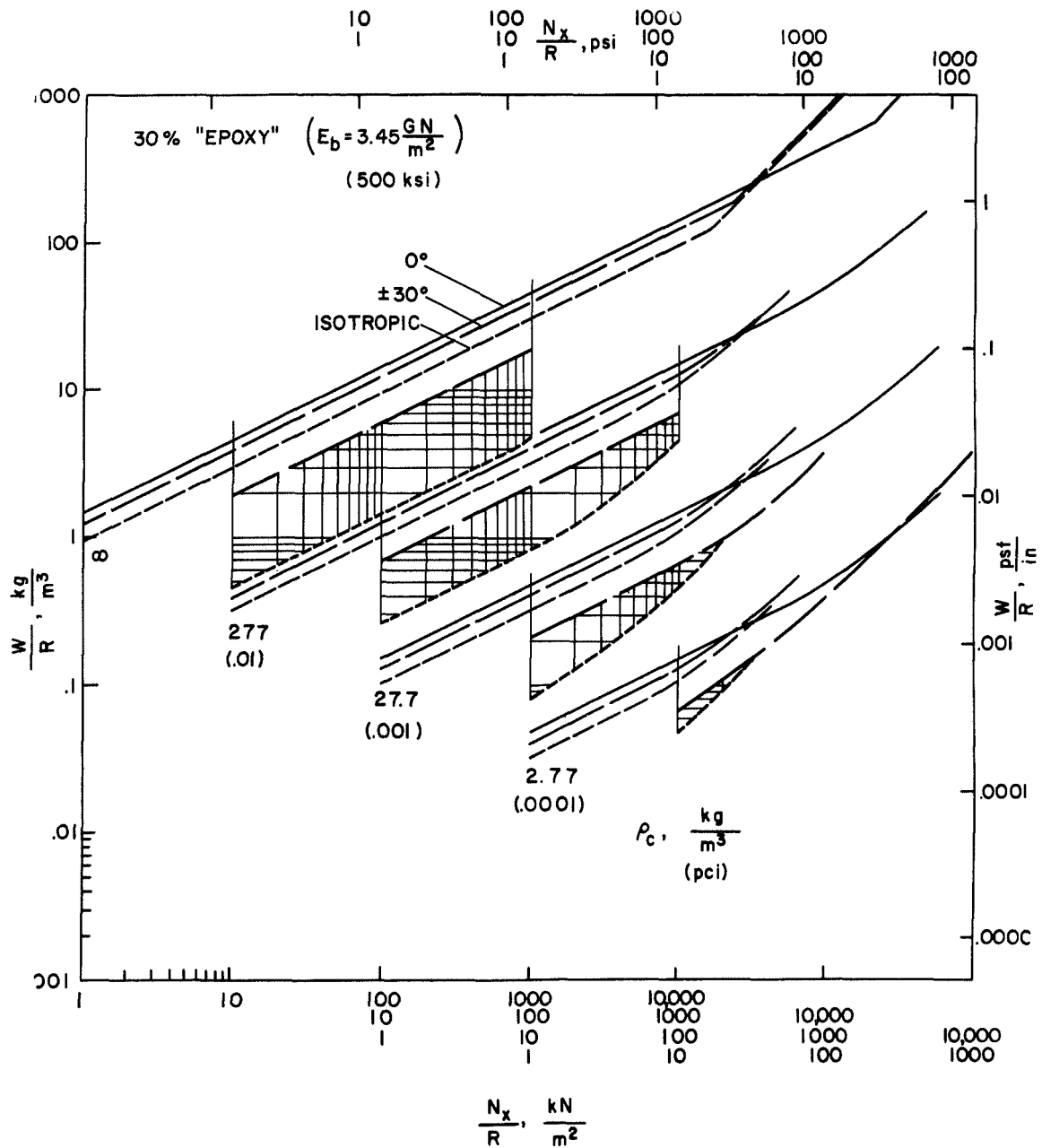


Figure 4. Shell Weight-Efficiencies of Steel Fibers in 30 Volume Percent Epoxy Binder, and Comparison with Metal Efficiencies

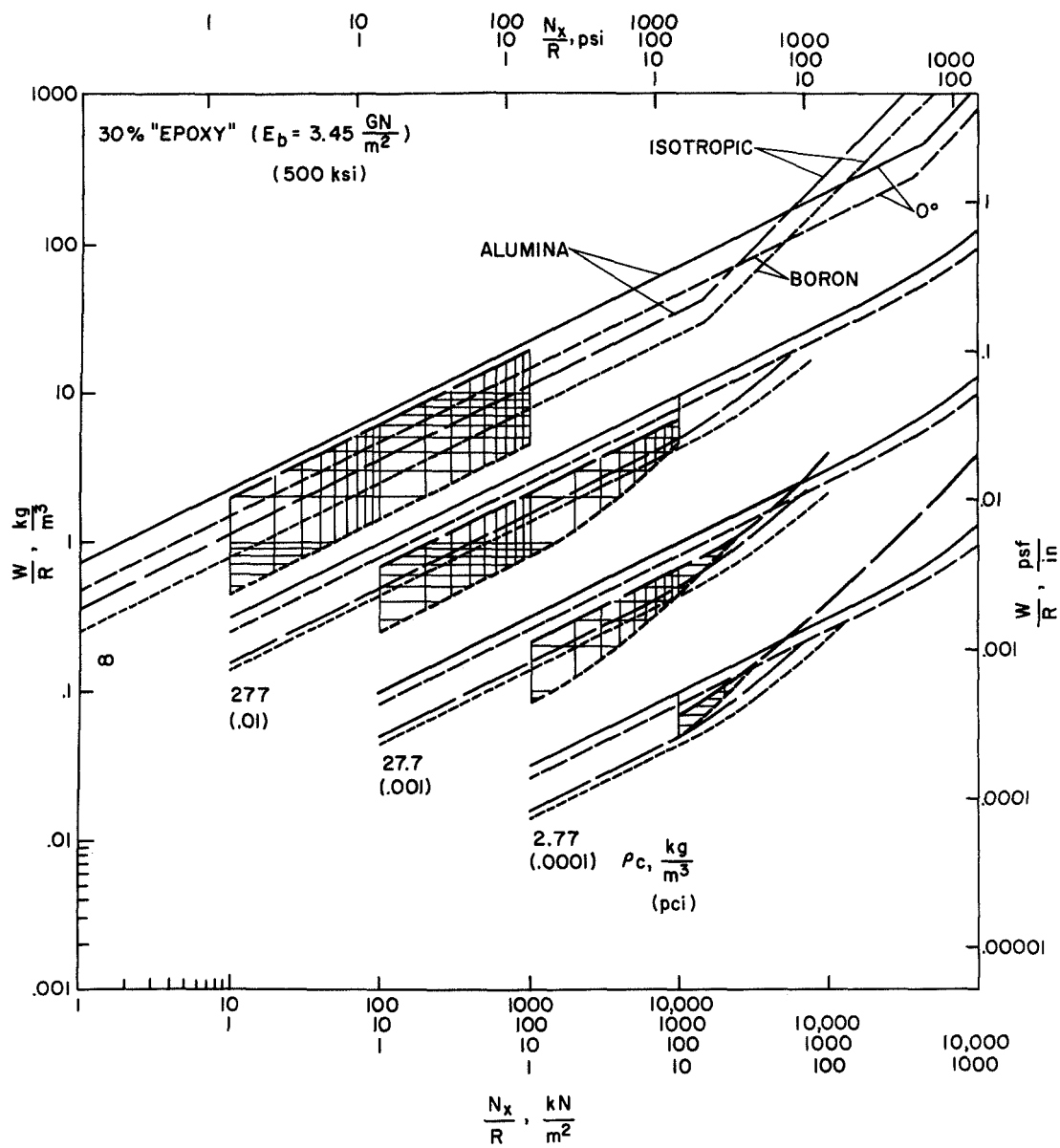


Figure 5. Shell Weight-Efficiencies of Boron and Alumina Fibers in 30 Volume Percent Epoxy Binder, and Comparison with Metal Efficiencies

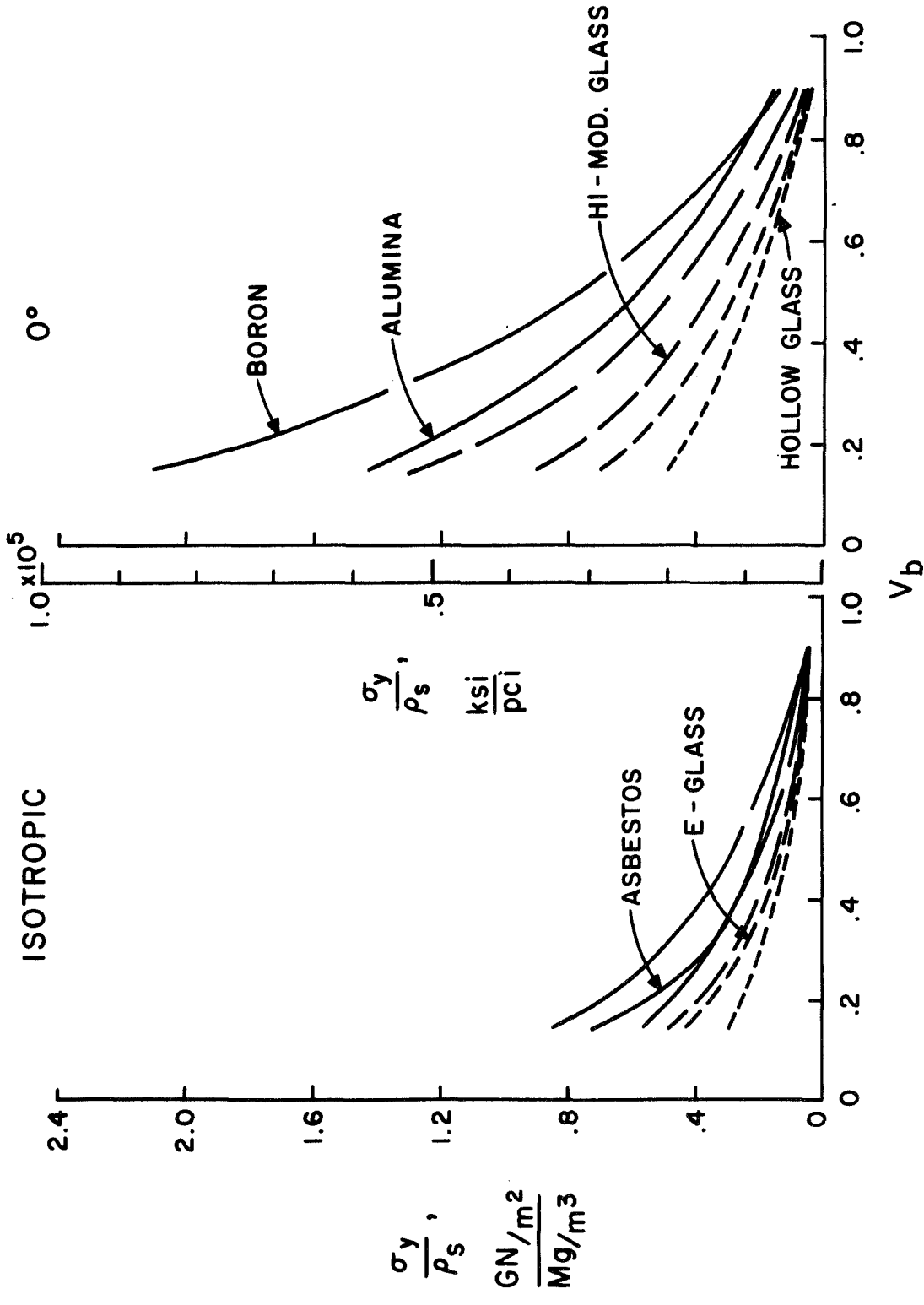


Figure 6. Analytical Estimates of Compressive Strength to Density Ratios of Epoxy Composites

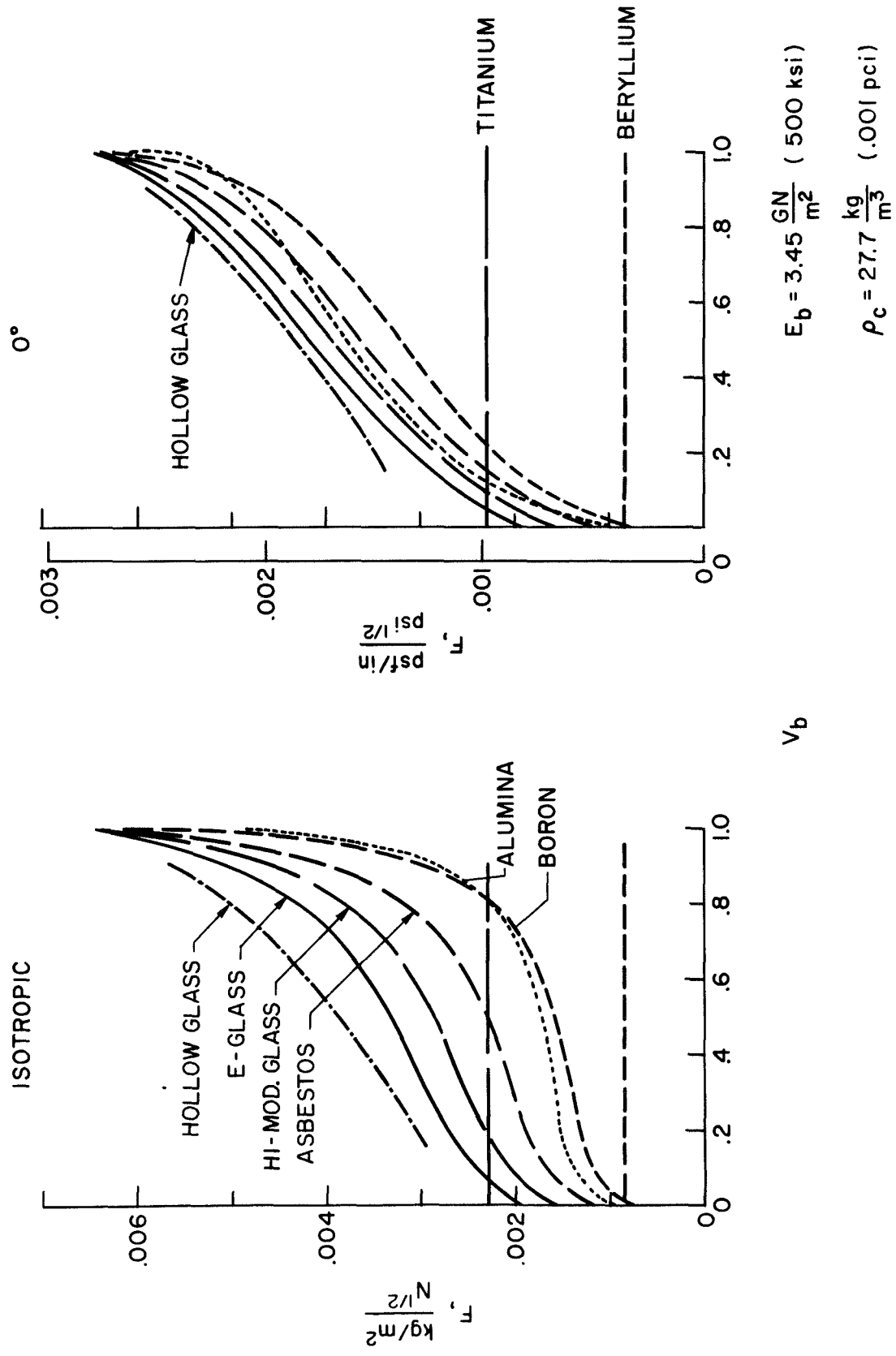


Figure 7. Elastic Buckling Efficiencies of Epoxy-Composite Sandwich Shells, and Comparison with Metal Efficiencies

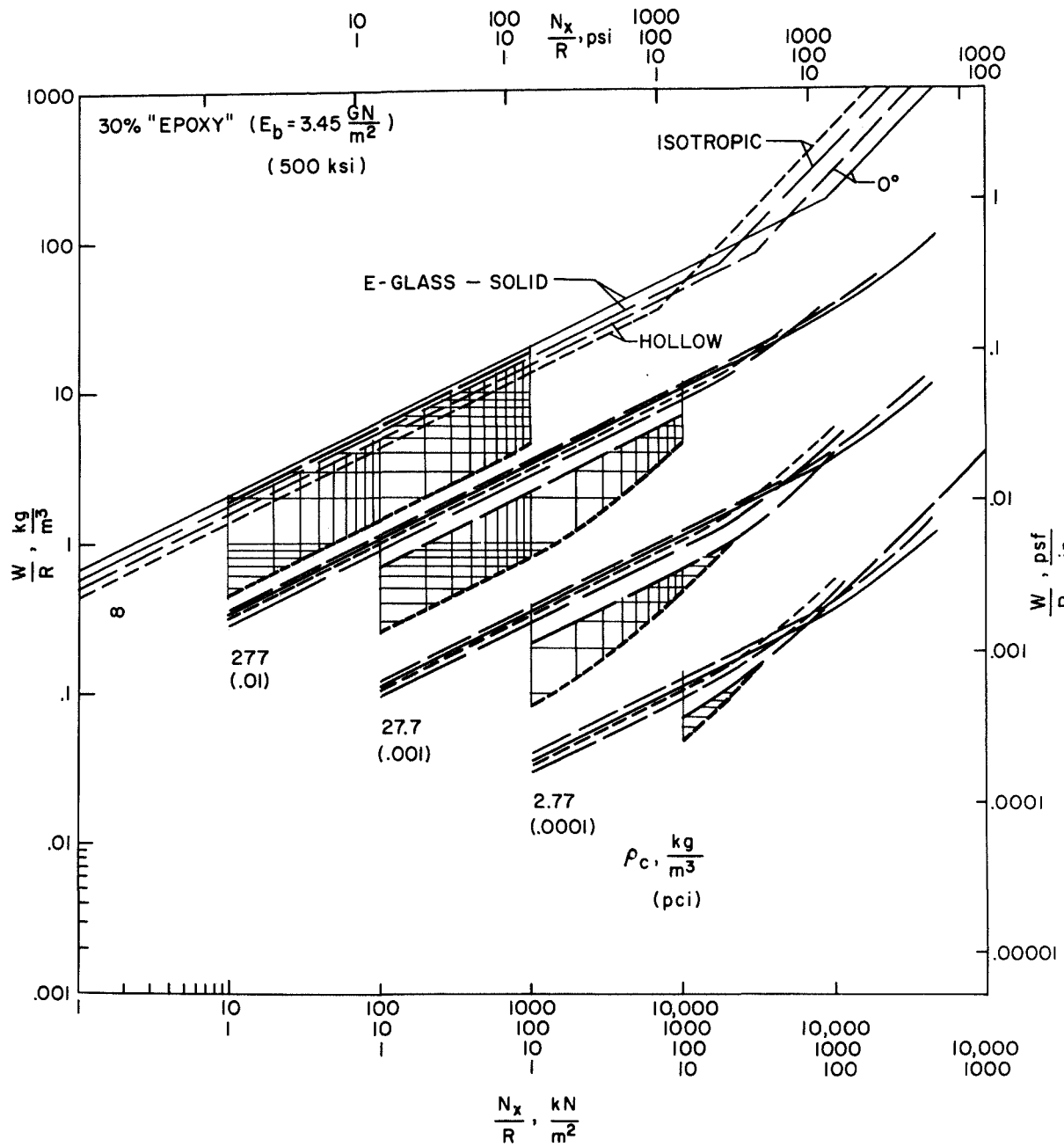


Figure 8. Shell Weight-Efficiencies of Solid and Hollow E-Class Fibers in 30 Volume Percent Epoxy Binder, and Comparison with Metal Efficiencies

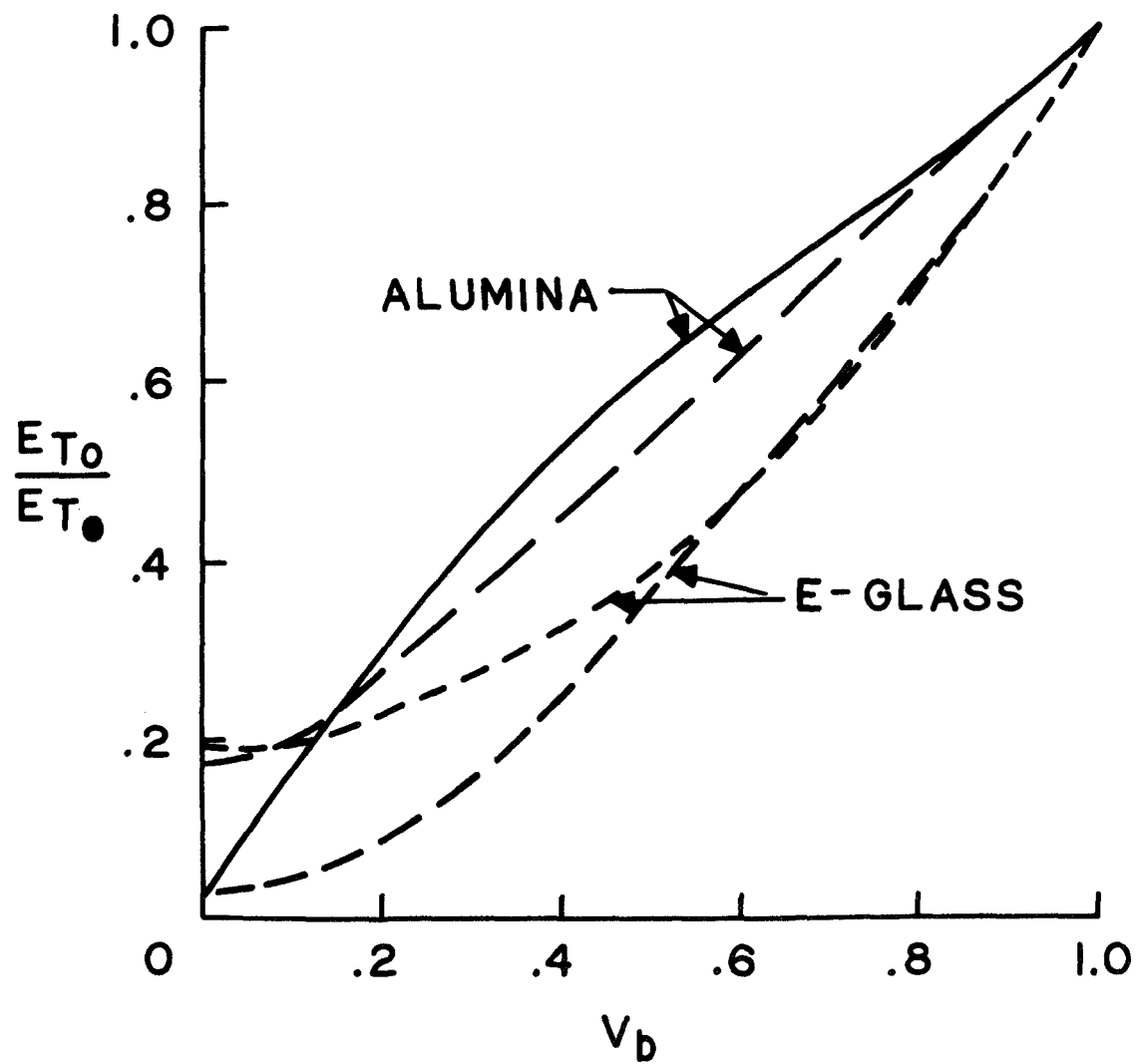


Figure 9. Ratio of Bounds for the Transverse Elastic Moduli of Hollow and Solid Fiber Composites

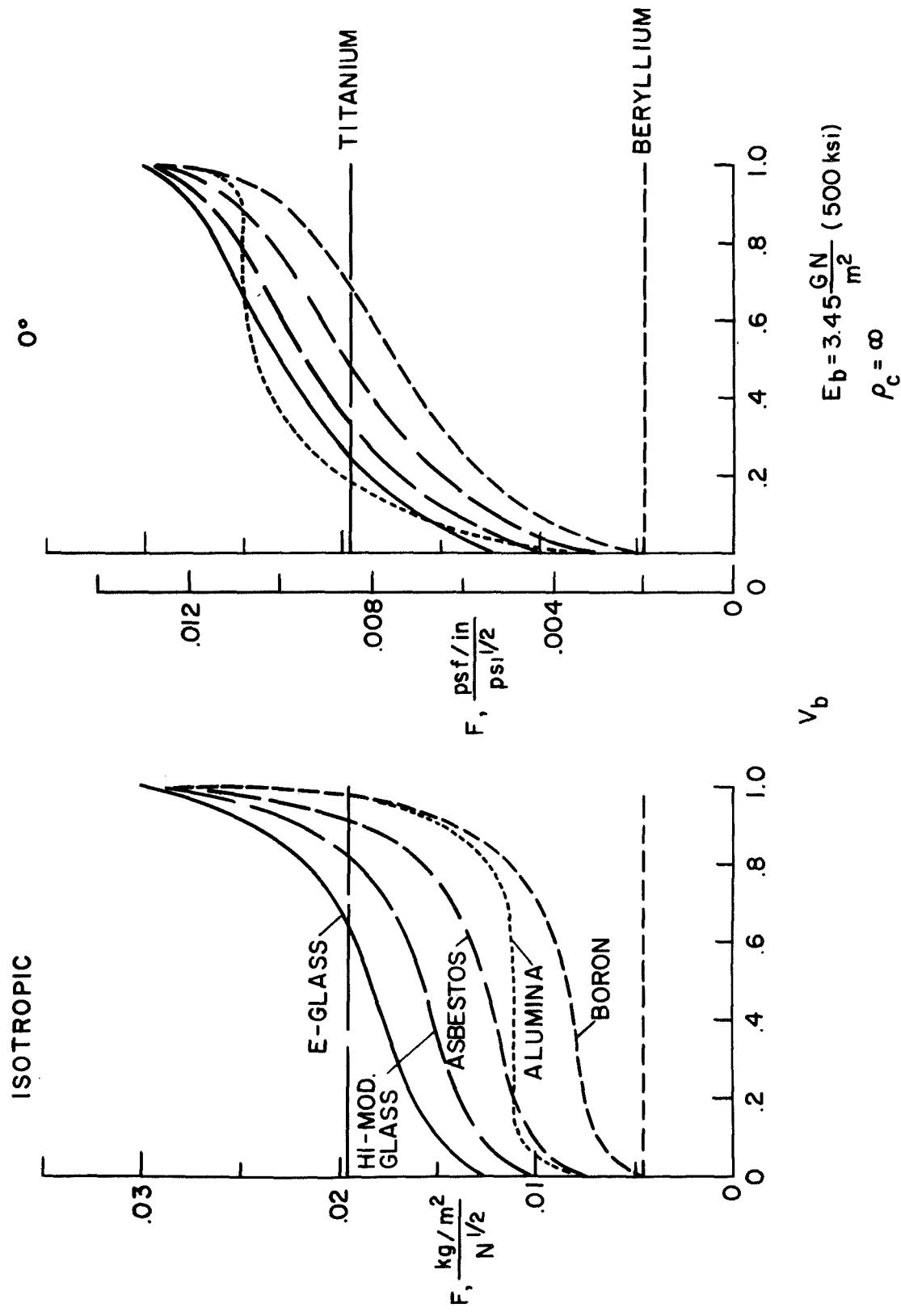


Figure 10. Elastic Buckling Efficiencies of Epoxy-Composite Monocoque Shells, and Comparison with Metal Efficiencies

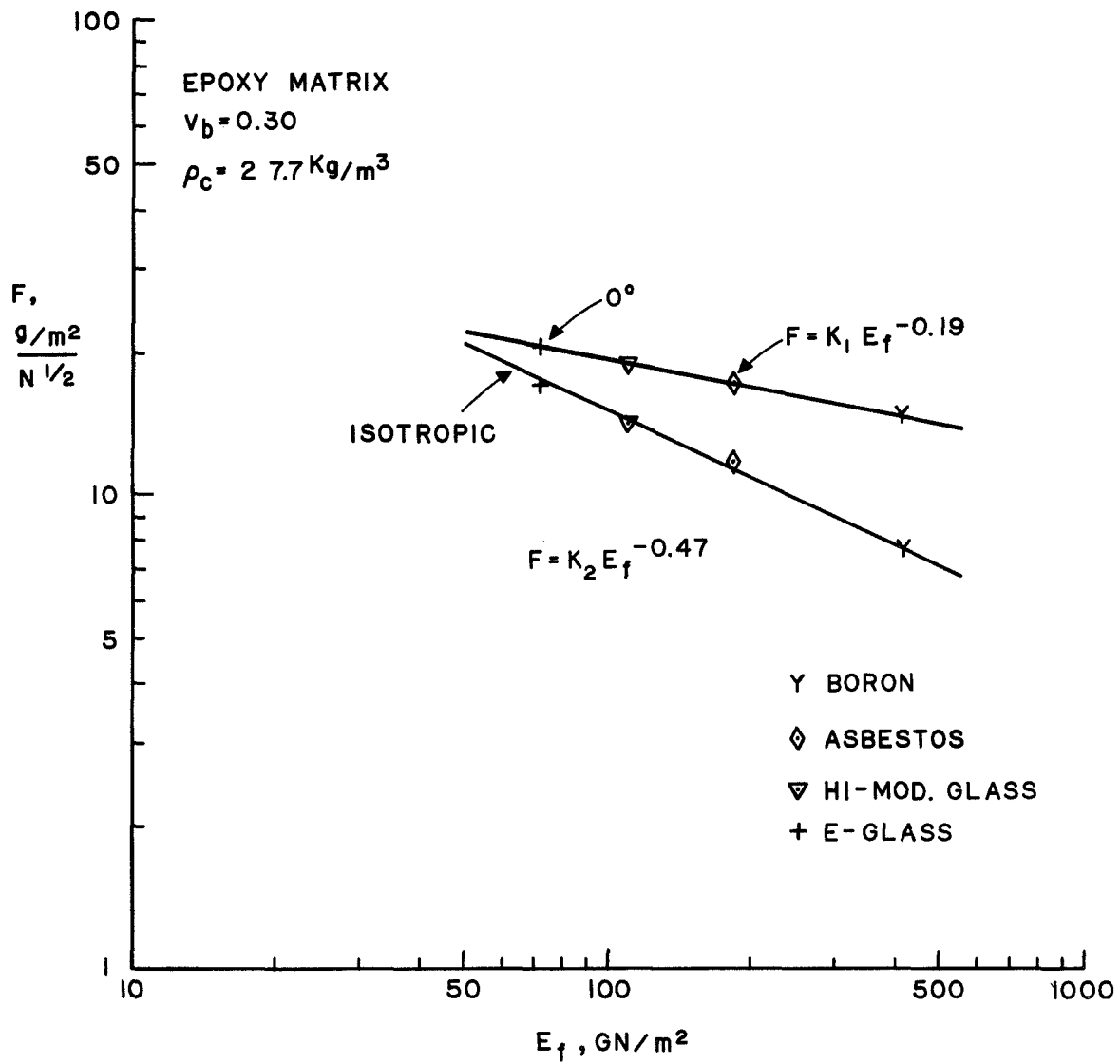


Figure 11. Elastic Buckling Efficiency of Epoxy Composite Sandwich Shells as a Function of Fiber Modulus



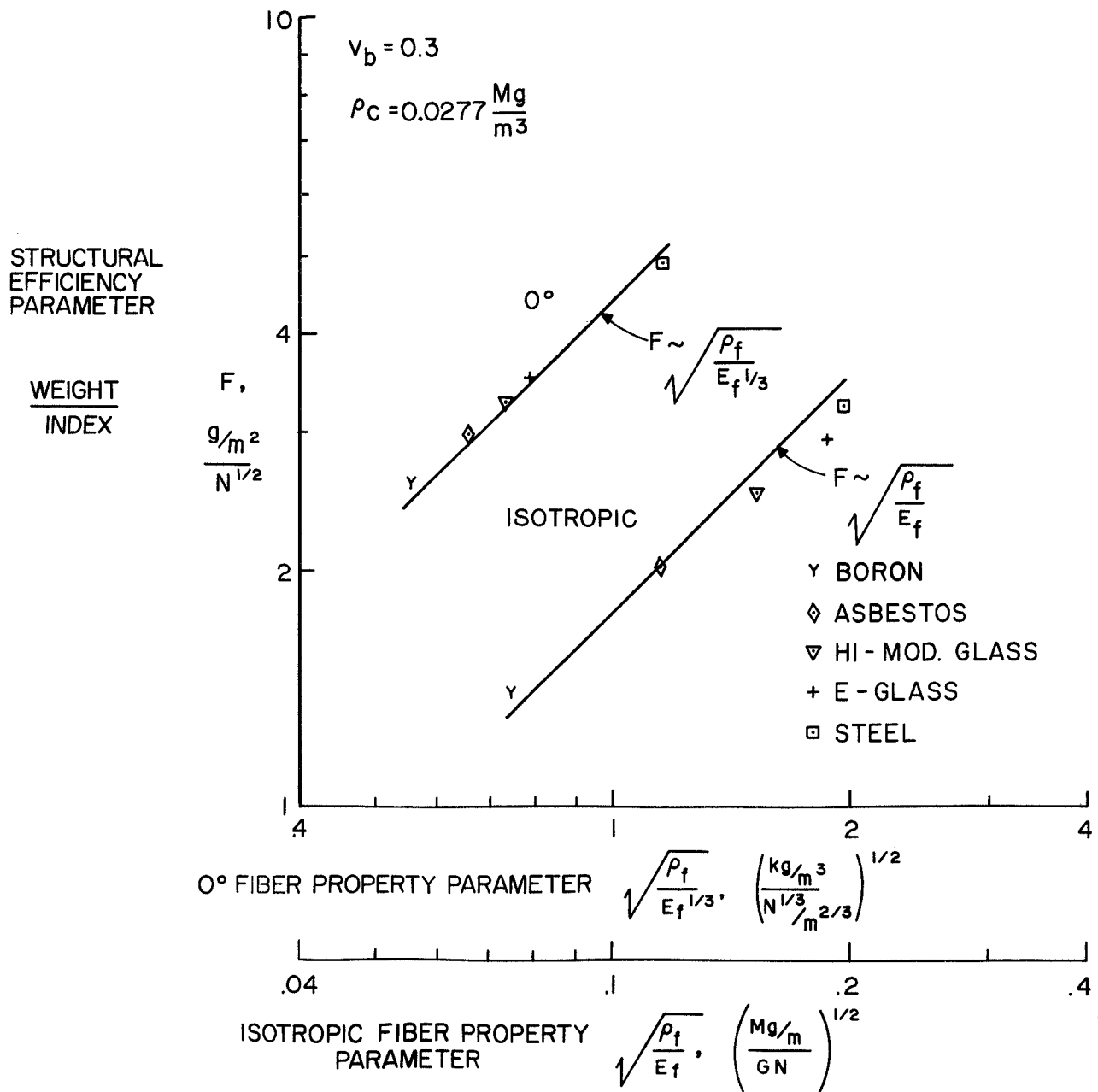


Figure 12. Elastic Buckling Efficiency of Fiber-Epoxy Composite Sandwich Shells as a Function of Fiber Properties

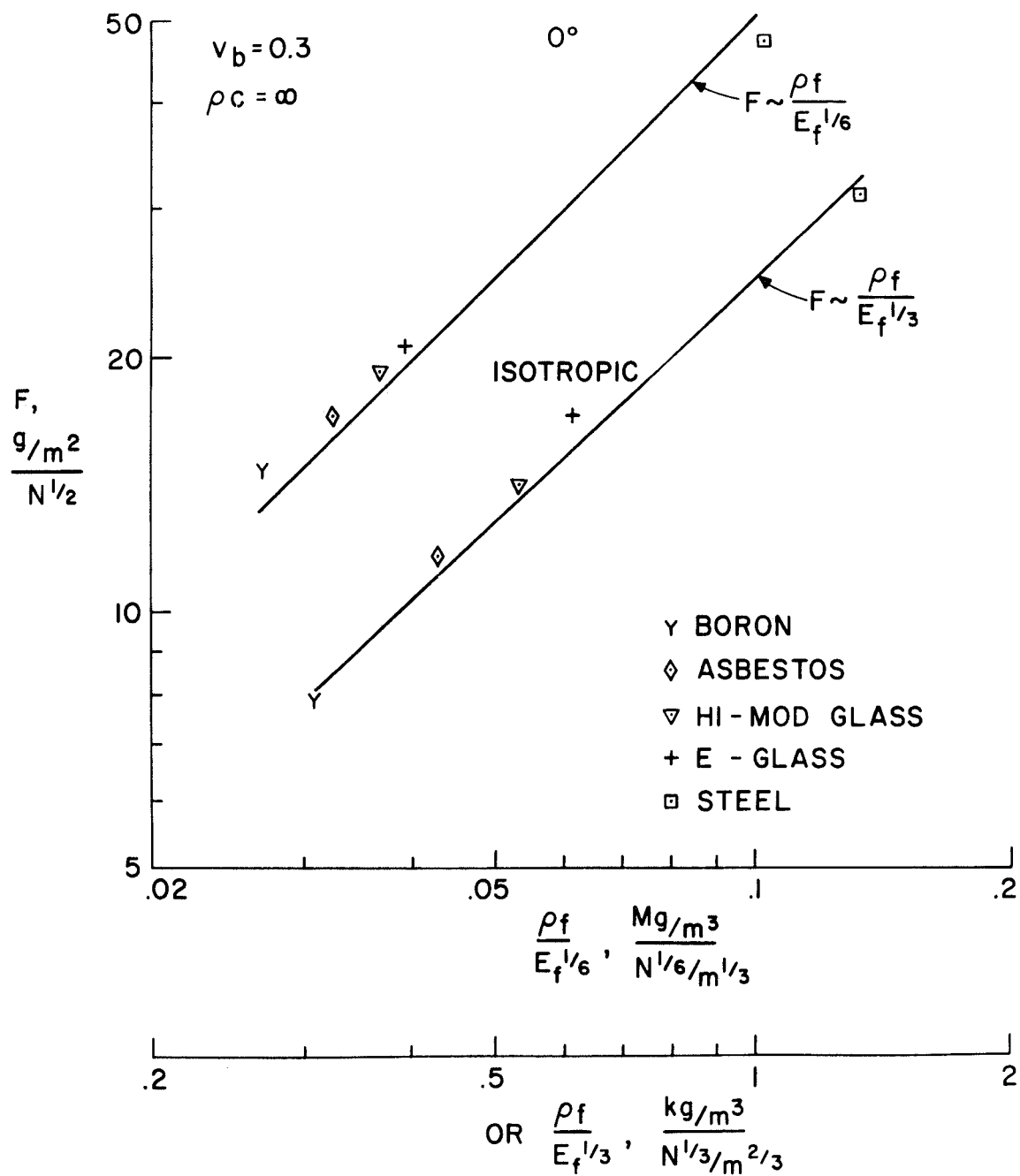


Figure 13. Elastic Buckling Efficiency of Fiber-Epoxy Composite Monocoque Shells as a Function of Fiber Properties

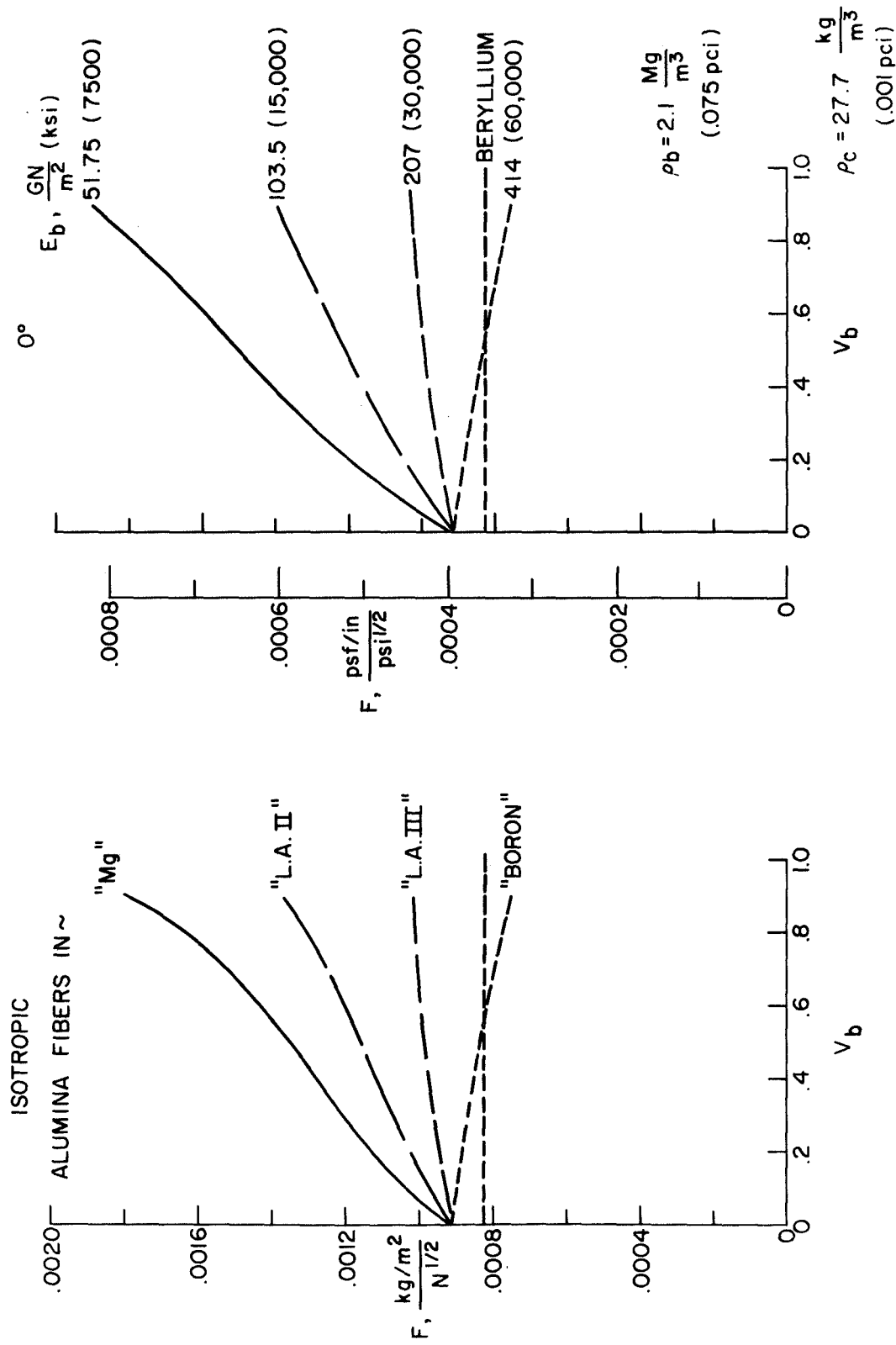


Figure 14. Elastic Buckling Efficiencies of Alumina-Fiber Reinforced Composite Sandwich Shells with Various Binders, and Comparison with Beryllium

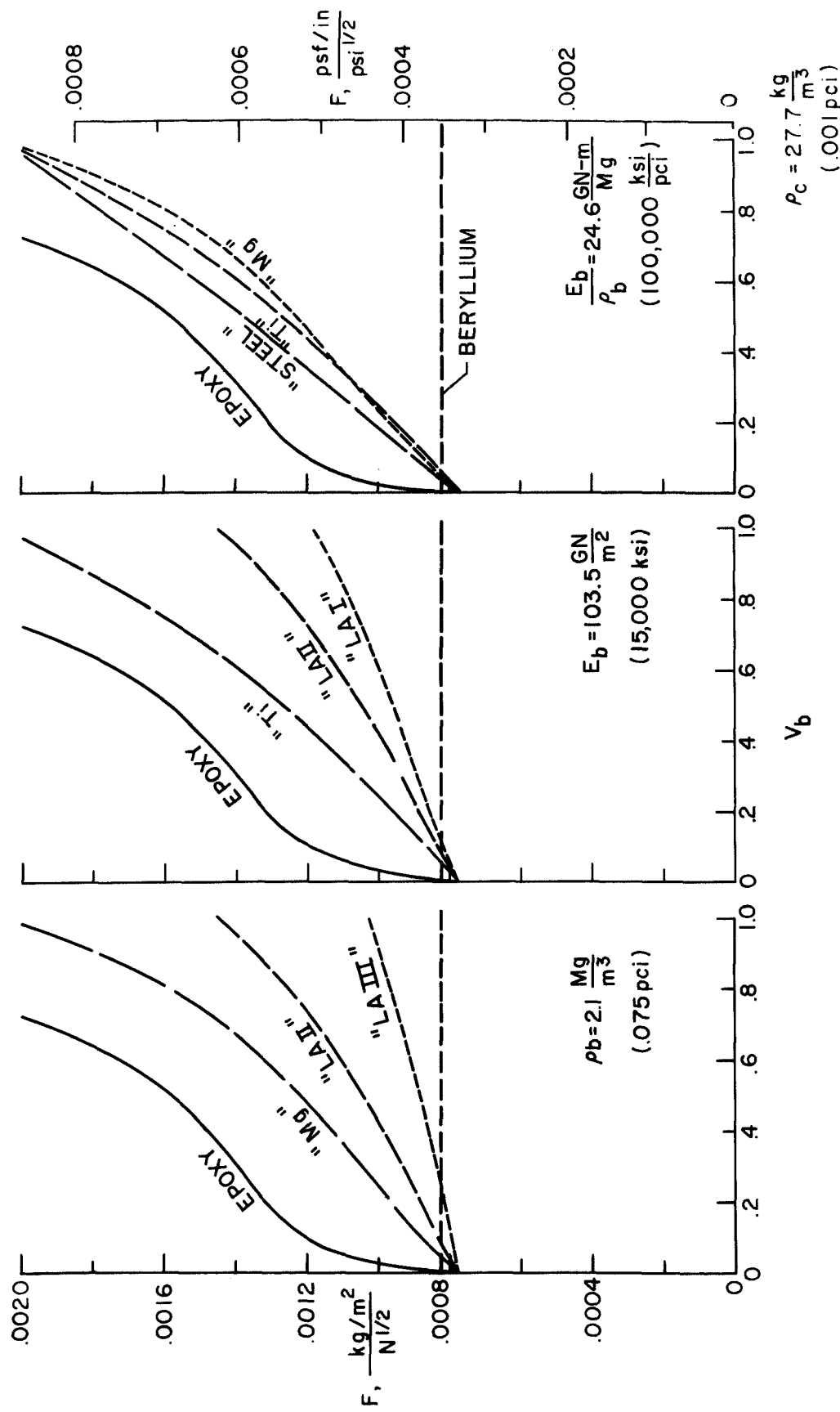


Figure 15. Effects of Variation in Binder Properties on Elastic Buckling Efficiency of  $\pm 30^\circ$ ,  $90^\circ$  Boron Filament Reinforced Composite Sandwich Shells

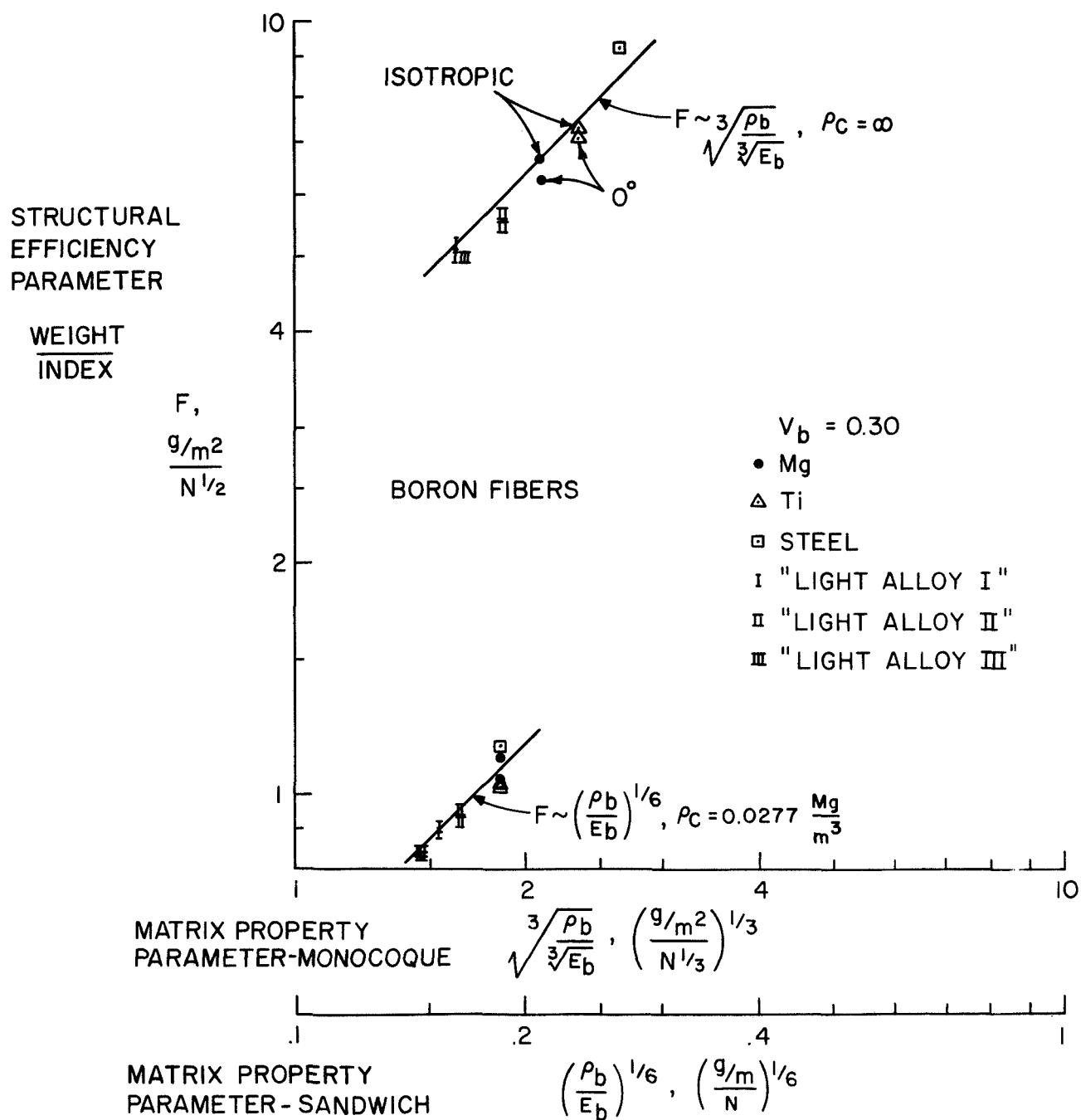


Figure 16. Elastic Buckling Efficiency of Fiber-Epoxy Composite Monocoque and Sandwich Shells as a Function of Binder Properties

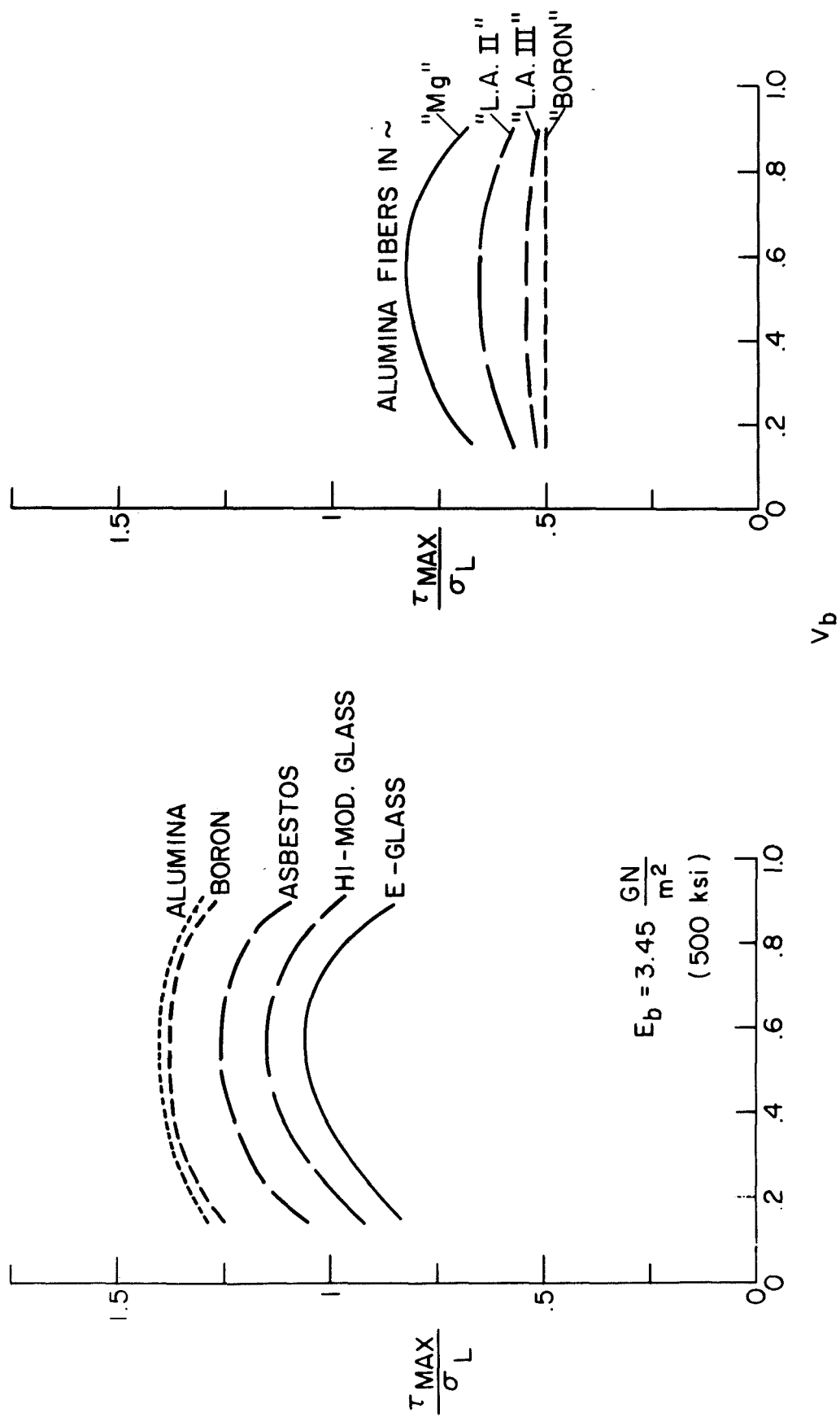


Figure 17. Maximum Shear Stresses Calculated for Various Filaments with Epoxy Binder, and for Alumina-Fibers in Various Binders (all Configurations Isotropic)

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